3. Calculating Solar Angles

These equations should be used keeping all of the angles in radians even though with some of the equations it does not matter whether degrees or radians are used.

3.1. Declination Angle

The declination angle is described in figure 1.5 and 1.6. The equation used to calculate the declination angle in radians on any given day is:

$$\delta = 23.45\frac{\pi}{180}\sin\left[2\pi\left(\frac{284+n}{365.25}\right)\right] \tag{3.1}$$

where:
δ = declination angle (rads);
n = the day number, such that n = 1 on the 1st January.

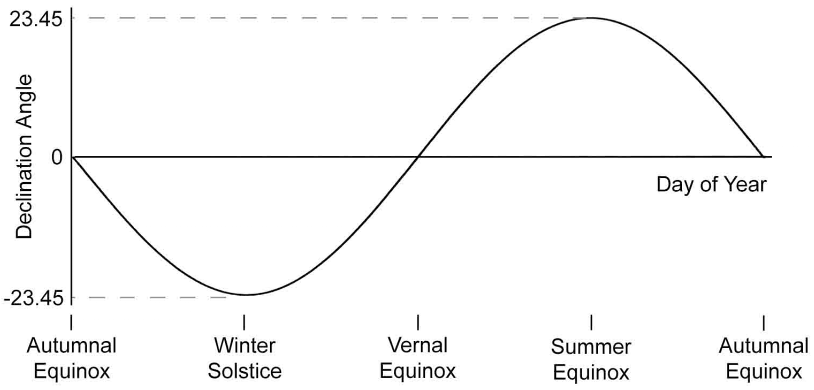

Figure 3.1: The variation in the declination angle throughout the year.

The declination angle is the same for the whole globe on any given day. Figure 3.1 shows the change in the declination angle throughout a year. Because the period of the Earth's complete revolution around the Sun does not coincide exactly with the calendar year the declination varies slightly on the same day from year to year.

3.2. The Hour Angle

The hour angle is described in figure 1.6 and it is positive during the morning, reduces to zero at solar noon and becomes increasingly negative as the afternoon progresses. Two equations can be used to calculate the hour angle when various angles are known (note that δ changes from day to day and α and A change with time throughout the day):

$$\sin\omega = -\frac{\cos\alpha\sin A_Z}{\cos\delta} \tag{3.2}$$

$$\sin\omega = \frac{\sin\alpha - \sin\delta\sin\phi}{\cos\delta\cos\phi} \tag{3.3}$$

where:
ω = the hour angle;
α = the altitude angle;

A_Z= the solar azimuth angle;
δ = the declination angle;
ϕ = observer's latitude.

Note that at solar noon the hour angle equals zero and since the hour angle changes at 15° per hour it is a simple matter to calculate the hour angle at any time of day. The hour angles at sunrise and sunset (ω_S) are very useful quantities to know. Numerically these two values have the same value however the sunrise angle is negative and the sunset angle is positive. Both can be calculated from:

$$\cos \omega_S = -\tan \phi \ \tan\delta \tag{3.4}$$

This equation is derived by substituting $\alpha = 0$ into equation 3.8. ω_S can be used to find the number of daylight hours (N) for a particular day using the next equation, where ω_S is in radians:

$$N = \frac{2\omega_S}{15} \times \frac{180}{\pi} \tag{3.5}$$

Note that there are always 4380 hours of daylight per year (non-leap years) everywhere on the globe.

For equation 3.4 beyond $\phi = \pm 66.55°$:
$(\tan\delta \tan\phi) \geq 1$ there is no sunset, i.e. 24 hours of daylight.
$(\tan\delta \tan\phi) \leq 1$ there is no sunrise, i.e. 24 hours of darkness.

If a surface is tilted from the horizontal the Sun may rise over its edge after it has rise over the horizon. Therefore the surface may shade itself for some of the day. The sunrise and sunset angles for a titled surface (ω_S') facing the equator (i.e. facing due south in the northern hemisphere) are given by:

$$\cos \omega_S' = -\tan(\phi - \beta) \tan \delta \tag{3.6}$$

where:
β = the angle of inclination of the surface from the horizontal.

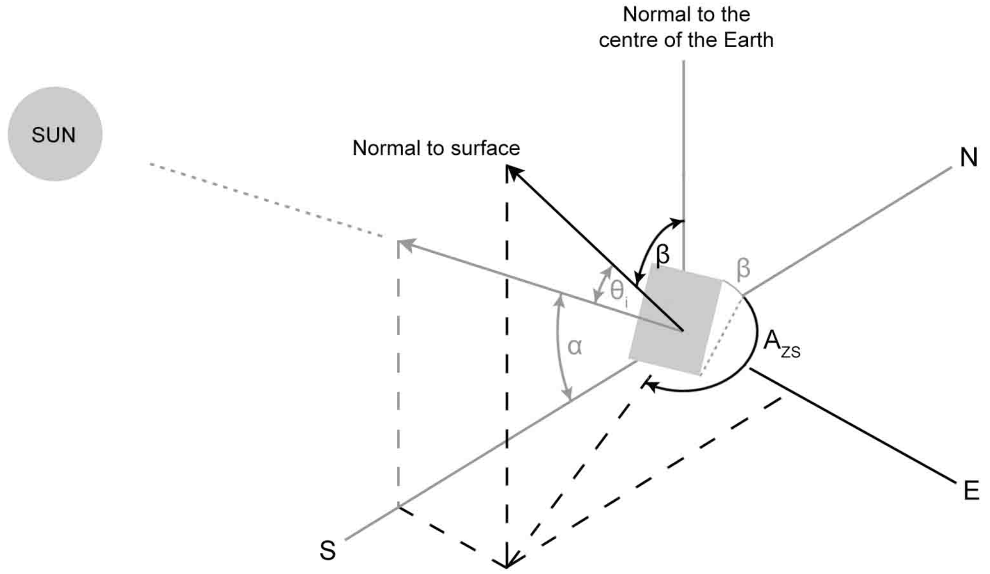

Figure 3.2: A tilted surface that is not facing the equator.

Equation 3.6 may give the sunrise angle for the tilted surface that indicates that the Sun rises over the edge of the surface before it has appeared over the horizon. This situation is obviously wrong and a check must be made to find the actual sunrise angle over the tilted plane (ω_0):

$$\omega_0 = \min\{\omega_S, \omega_s'\}$$

Note that for a titled surface facing the equator the sunrise and sunset angles are still numerically equal with the sunrise angle being positive and the sunset angle being negative. When a surface is inclined from the horizontal but not facing the equator, calculating the sunrise and sunset angles over the edge of the surface is complex. Such a surface is shown in figure 3.2. For such a surface the sunrise and sunset angles (ω_S'') will not be numerically equal and the following procedure must be followed:

$$\omega_S' = \cos^{-1}\left[\frac{ab \pm \sqrt{a^2 - b^2 + 1}}{a^2 + 1}\right] \tag{3.7}$$

Where:

$$a = \frac{\cos\phi}{\sin A_{ZS} \tan\beta} + \frac{\sin\phi}{\sin A_{ZS}}$$

$$b = \tan\delta\left[\frac{\cos\phi}{\tan A_{AZ}} - \frac{\sin\phi}{\sin A_{AZ} \tan\beta}\right]$$

Equation 3.7 gives two solutions because of the ± sign, one is the sunset angle and the other is the sunrise angle. Then ω_0 is checked as before:

$$\omega_0 = \min\{\omega_S, \omega_s''\}$$

3.3. The Altitude Angle

The altitude angle (α) is described in figure 1.7 and can be calculated from:

$$\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\omega\cos\phi \tag{3.8}$$

3.4. The Azimuth Angle

The azimuth angle is described in figure 1.7 and can be calculated from the following equation:

$$\sin\alpha = \frac{\sin\omega\cos\delta}{\sin\theta_Z} = \frac{\sin\omega\cos\delta}{\cos\alpha} \tag{3.9}$$

The azimuth angle at sunrise (A_{SR}) can be calculated from:

$$\sin A_{SR} = -\sin\omega_S\cos\delta \tag{3.10}$$

Note that that A_Z is normally measured clockwise from due north although sometimes due south is taken to be zero and A_Z is then negative when the Sun is east from this point and positive when the Sun is west of this point

3.5. Angle Of Incidence

The angle of incidence (θ_i) of the Sun on a surface tilted at an angle from the horizontal (β) and with any surface azimuth angle (A_{ZS}) (figure 3.2) can be calculated from (when A_{ZS} is measured clockwise from north):

$$\begin{aligned}\cos\theta_i &= \sin\delta\sin\phi\cos\beta + \sin\delta\cos\phi\sin\beta\cos A_{ZS} + \cos\delta\cos\phi\cos\beta\cos\omega \\ &\quad - \cos\delta\sin\phi\sin\beta\cos A_{ZS}\cos\omega - \cos\delta\sin\beta\sin A_{ZS}\sin\omega\end{aligned} \tag{3.11}$$

This horrible equation can be simplified in a number of instances. When the surface is flat (i.e. horizontal) $\beta = 0$, $\cos\beta = 1$, $\sin\beta = 0$. Therefore equation 3.11 becomes:

$$\cos\theta_i = \cos\theta_Z = \cos\delta\cos\phi\ \cos\omega + \sin\delta\sin\phi \tag{3.12}$$

When the surface is tilted towards the equator (facing south in the northern hemisphere):

$$\cos\theta_i = \cos\delta\cos(\phi-\beta)\cos\omega + \sin\delta\sin(\phi-\beta) \tag{3.13}$$

Note that if $\theta_i > 90°$ at any point the Sun is behind the surface and the surface will be shading itself.